

2013年度日本政府(文部科学省)奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR JAPANESE
GOVERNMENT (MONBUKAGAKUSHO) SCHOLARSHIPS 2012

学科試験 問題

EXAMINATION QUESTIONS

(高等専門学校留学生)

COLLEGE OF TECHNOLOGY STUDENTS

数 学

MATHEMATICS

注意 ☆試験時間は60分

PLEASE NOTE : THE TEST PERIOD IS 60 MINUTES.

MATHEMATICS

Nationality		No.		Marks	
Name	(Please print full name, underlining family name)				

1 Answer all questions and write your answers in the boxes provided.

1) Let $x = \frac{3 + \sqrt{5}}{3 - \sqrt{5}}$ and $y = \frac{3 - \sqrt{5}}{3 + \sqrt{5}}$. Calculate $x^3 + y^3$.

2) Solve the equation $x^3 + 5x^2 + 8x + 6 = 0$.

3) Solve the equation $3^{2x+1} + 2 \cdot 3^x - 1 = 0$.

4) Solve the equation $\cos^2 x - \sin x \cos x = 1$ ($0 \leq x < 2\pi$).

5) Solve the inequality $x^2 - 4x + 2 < 0$.

6) Solve the inequality $(\log_2 x)^2 - 2 \log_4 x^3 + 2 < 0$.

7) Let $\vec{a} = (2, 1, -3)$, $\vec{b} = (1, 0, -2)$ and $\vec{c} = \vec{a} + t\vec{b}$. Find the value of the constant t which satisfies $\vec{c} \perp \vec{a}$.

8) Calculate the area of the triangle OAB with vertices $O(0, 0, 0)$, $A(3, 1, 4)$, $B(-1, 4, 3)$.

9) A sequence $\{a_n\}$ satisfies the following conditions. Calculate $\lim_{n \rightarrow \infty} a_n$.

$$a_1 = 3, \quad a_{n+1} = \frac{2}{3}a_n + \frac{1}{4} \quad (n = 1, 2, 3, \dots)$$

10) Calculate $\lim_{x \rightarrow \infty} (\sqrt{x^2 + 3x + 1} - x)$.

11) Find the value of the constant k for which the line $y = 2x + 1$ is a tangent to the curve $y = x^3 + kx + 3$.

12) Find the value of x which minimizes $f(x) = x - \sqrt{1 - x^2}$.

2 Let $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $O = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$. Answer the following questions and write your answers in the boxes provided.

1) Let $A = \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & a \\ b & 2 \end{pmatrix}$. Find the values of a and b which satisfy $AB = O$.

$$a =$$

$$b =$$

2) Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 5 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 1 \\ 8 & 2 \end{pmatrix}$. Find the matrix X satisfying $AX = B$.

$$X =$$

3) Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ satisfying $A^2 + A - 6I = O$ and $a + d > 0$. Find the values of $a + d$ and $ad - bc$.

$$a + d =$$

$$ad - bc =$$

3 Answer all questions and write your answers in the boxes provided.

1) Calculate $\int_0^{\pi} \sin^2 t \, dt$.

2) Calculate $\int_0^{\pi} t \sin^2 t \, dt$.

3) Find the function $f(x)$ which satisfies $f(x) = x + \frac{1}{\pi} \int_0^{\pi} f(t) \sin^2 t \, dt$.